Finite Math - J-term 2019

Lecture Notes - 1/30/2019

Homework

- Section 8.2 - 7, 9, 11, 13, 15, 16, 17, 19, 25, 26, 27, 28, 33, 35, 37, 28

Section 8.1 - Sample Spaces, Events, and Probability

Equally Likely Assumption. When we were talking about assigning probabilities of 0.5 to heads and 0.5 to tails for flipping a coin, and a probability of $\frac{1}{6}$ for any number to come up when rolling a 6-sided die, we are making an assumption on the probabilities of the experiment called an *equally likely assumption.* In an ideal case, this poses no risk, but as we talked about, a coin or a die may not necessarily be "fair."

Generally, if the sample space is

$$S = \{e_1, e_2, ..., e_n\},$$

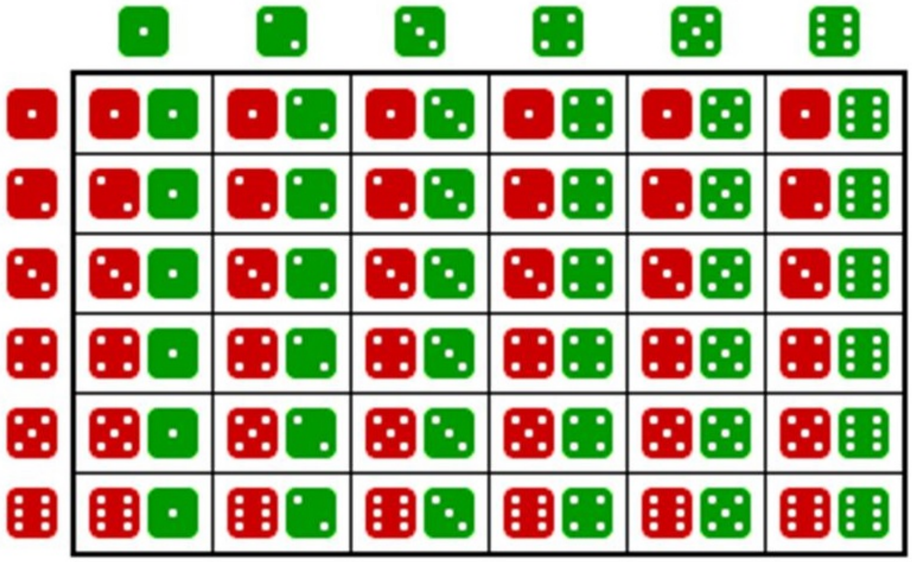
we assign to each e_i a probability of $\frac{1}{n}$ since there are n possible outcomes and we want each of them to be equally likely. This gives us the following theorem

Theorem 1 (Probability of an Arbitrary Event under an Equally Likely Assumption)**.** *If we assume that each simple event in a sample space S is equally likely to occur, then the probability of an arbitrary event E in S is given by*

$$P(E) = \frac{n(E)}{n(S)},$$

the number of elements in E divided by the number of elements in S.

We saw this theorem in action when we found the theoretical probabilities for rolling a number on a pair of dice. Recall our sample space for rolling a pair of dice:



or written in a more mathematical way

SECOND DIE

FIRST DIE	⚀	⚁	⚂	⚃	⚄	⚅
⚀	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
⚁	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
⚂	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
⚃	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
⚄	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
⚅	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example 1. *Refer back to the example where we roll two dice. If we assume that every simple event is equally likely:*

(a) *What is the probability of a simple event happening?*

(b) *What are the possible numbers that the two dice could add up to?*

(c) *What are the probability of each of the events in part (b) happening?*

Empirical Approach. In an empirical approach to probability, we run the experiment several times, and assign probabilities according to the frequency which with outcomes occurred. For example, if we flip a coin 1000 times and get 373 heads and 627 tails, we would be tempted to assign probabilities as

$$P(H) = \frac{373}{1000} \qquad P(T) = \frac{623}{1000}$$

since it reflects the results of an extensive experiment.

The number of times an event E occurs in an experiment is called the *frequency* of the event, and is denoted $f(E)$. If the experiment has n trials (in the example above, $n = 1000$ since there was 1000 coin flips), the *relative frequency* of the event E in n trials is the number $\frac{f(E)}{n}$. We can define the *empirical probability* of E, which we will denote by $P(E)$, by the number that $\frac{f(E)}{n}$ approaches as n gets larger and larger. The reasoning behind this is that given an infinite number of runs of the experiment, the relative frequency should reflect the actual probabilities, and generally, this number becomes a better and better approximation as the number of runs of the experiment increases. For any finite number n we plug in, $\frac{f(E)}{n}$ is an *approximate empirical probability* of the event E.

Experiment Time! In the previous example, we came up with the probabilities of rolling a given number as the sum of two dice. Now we will take an empirical approach to test this and see if those numbers hold up! Roll your pair of dice 50 times and record the sums of your rolls (I suggest making a table with the possible outcomes and just using tally marks). Compute the approximate empirical probabilities for the different sums of dice based off your experiment. How does it compare to the theoretical probabilities with the equally likely assumption?

Section 8.2 - Union, Intersection, and Complement of Events; Odds

Union and Intersection.

Definition 1 (Union and Intersection of Events). *If A and B are two events in a sample space S (i.e., $A \subset S$ and $B \subset S$), we define two new events:*

- *The* event A or B *is the union* $A \cup B$
- *The* event A and B *is the intersection* $A \cap B$

Example 2. *Suppose we are rolling a single fair die (each number is equally likely), so our sample set is*

$$S = \{1, 2, 3, 4, 5, 6\}.$$

(a) *What is the probability of rolling a number which is even and divisible by 3?*

(b) *What is the probability of rolling a number which is even or divisible by 3?*

Example 3. *Suppose we are rolling a single fair die, so our sample set is*

$$S = \{1, 2, 3, 4, 5, 6\}.$$

(a) *What is the probability of rolling a number which is odd and prime?*

(b) *What is the probability of rolling a number which is odd or prime?*

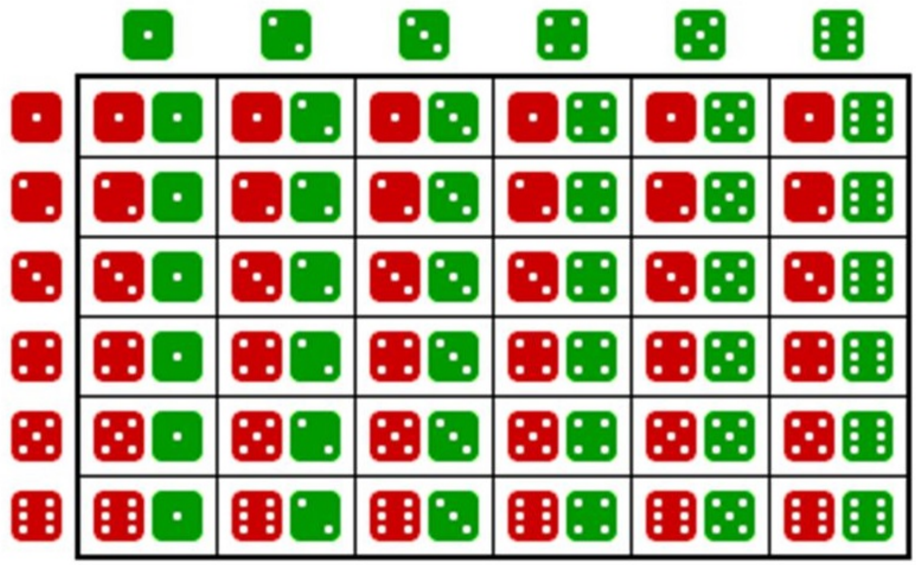
Theorem 2 (Probability of the Union of Two Events)**.** *For any events A and B,*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Events A and B are called *mutually exclusive* if $A \cap B = \varnothing$.

Remark 1. *Note the similarity to the formula for the addition principle:*

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$



Example 4. *Suppose that two fair dice are rolled.*

(a) *What is the probability that a sum of 7 or 11 turns up?*

(b) *What is the probability that both dice turn up the same or that a sum less than 5 turns up?*

Example 5. *What is the probability that a number selected at random from the first* 500 *positive integers is:*

(a) *divisible by* 3 *or* 4*?*

(b) *divisible by* 4 *or* 6*?*

Complement of an Event. Let $S = \{e_1, e_2, ..., e_n\}$ be a sample space and let E be some event. Then the set E' is also an event and since $E \cap E' = \varnothing$ and $E \cup E' = S$, then we have

$$P(S) = P(E \cup E') = P(E) + P(E') = 1.$$

So it follows that

$$P(E) = 1 - P(E') \qquad \textit{and} \qquad P(E') = 1 - P(E).$$

If E is an event, then E' is "the event that E *does not* happens."
As a simple example, if E is the event that it snows outside and $P(E) = .35$, then E' is the event that it does not snow and $P(E') = .65 = 1 - .35$.

Example 6. *A shipment of* 45 *precision parts, including* 9 *that are defective, is sent to an assembly plant. The quality control division selects* 10 *at random for testing and rejects the entire shipment if* 1 *or more in the sample are found to be defective. What is the probability that the shipment will be rejected?*

Example 7. *Assume the same set up from the last problem, but only* 40 *parts are shipped,* 8 *of which are defective. What is the probability of rejection this time?*

Example 8. *Let's assume there are* 365 *days in a year (sorry anyone born on February 29th). In a group of* n *people, what is the probability that at least* 2 *people have the same birthday? What value of* n *is required for the probability to be at least* 50%*?* 90%*?* 99%*?*